

## The Mathematical Content Knowledge and Attitudes of New Zealand Pre-service Primary Teachers

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This paper presents data on the mathematical content knowledge and attitudes of pre-service primary teacher education students. The assessment consisted of nine tasks, including 2-digit computations and proportional reasoning. Students rated their liking for mathematics at three time points: primary, secondary, and when assessed. Fewer than half the students liked mathematics currently. Those with positive attitudes tended to perform well on mathematics tasks, but some low scorers were positive and some high scorers were negative about mathematics. Most students used algorithmic procedures to solve problems and several consistent misconceptions were identified. Performance was noticeably poor on adding common fractions and converting fractions to percentages using knowledge of common factors. The implications of these findings for Initial Teacher Education (ITE) providers are presented.

It is often claimed that teachers cannot teach what they do not know themselves. The importance of primary teachers having sufficient knowledge of mathematical content to meet the challenging demands of teaching mathematics to children is no longer in dispute. There has been substantial research on the mathematical content knowledge of pre-service teachers and efforts to strengthen this knowledge (e.g., Callingham & Beswick, 2011; Mays, 2005; Watson, 2011).

In recent years, governments and education systems have begun to show concern about the issue of teacher quality (e.g., Australian Council of Deans of Education [ACDE], 2012; Australian Institute for Teaching and School Leadership [AITSL], 2011; Commonwealth of Australia, 2008; Department for Education, 2010; New Zealand Government, 2010; Organisation for Economic Cooperative Development [OECD], 2005). According to the Commonwealth of Australia (2008, p. 21), one of the main issues associated with Australia's mathematics and numeracy teaching workforce is that "primary teachers are not being adequately prepared for teaching numeracy and mathematics". Concern has been expressed about the need to attract high-achieving students into teacher education, with the goal of recruiting the top 30% of the population. There has also been discussion about whether assessment of mathematical content knowledge should be mandatory on entry to and exit from an initial teacher education (ITE) program. According to the ACDE report, "the teaching of mathematics depends not only on a [teacher's] mathematical content knowledge, but also on their attitudes to teaching mathematics and their understanding of mathematics pedagogy" (2012, p. 14).

### *Attitudes towards Mathematics*

Not only do prospective teachers need a good understanding of the mathematics they will eventually teach, but a positive attitude towards mathematics is also important (AITSL, 2011, McGinnis et al., 2002; Southwell, White, Way, & Perry, 2005). As Lomas, Grootenboer, and Attard (2012) have pointed out, there is relatively little recent research on attitudes towards mathematics. Biddulph (1999) reported that only one-quarter of New Zealand ITE students surveyed (halfway through their first year, before starting mathematics education) were positive, and more than half were distinctly negative about mathematics. Of students nearing the end of a Bachelor of Teaching program, just over half (57%) were positive about mathematics, with only 16 per cent expressing negative feelings (Young-Loveridge, 2010). When asked about their attitude towards the prospect of teaching mathematics, two-thirds were positive, and only 17 per cent were negative. The relationship between general attitudes towards mathematics and attitudes towards teaching mathematics seems to be complex and nuanced. Some students were positive about mathematics, possibly having experienced success with school mathematics, but were anxious about the responsibility of teaching others, and hence negative about the prospect of teaching mathematics. Others were negative about mathematics because of unpleasant experiences learning school mathematics, but looked forward to teaching mathematics more effectively than their own teachers, so were positive about the prospect of teaching mathematics.

Evidence shows that teachers' attitudes influence their classroom practices, and this impacts on the attitudes and learning of their students. Southwell et al. (2005) identified two independent factors, insecurity and confidence, contributing to both attitudes towards mathematics and to teaching mathematics. Recognition of the connection between attitudes and beliefs about mathematical content knowledge and reform-based mathematics pedagogy in ITE programs have led to initiatives to improve the attitudes and beliefs of prospective teachers, with encouraging results (e.g., McGinnis et al., 2002).

### *The Impact of Mathematics Education Reform*

Mathematics education reform has been under way for several decades. Skemp's (1972) idea of *instrumental* versus *relational* understanding epitomises the contrast of traditional approaches to mathematics teaching focusing on the memorisation of rules, facts, and procedures to accumulate correct answers, with more recent reform-oriented approaches. The latter emphasize making sense, building conceptual understanding, recognising the connectedness among concepts, together with thinking and reasoning mathematically. Although it is 40 years since Skemp's original work, change in the teaching of mathematics has been extremely slow. Evidence shows that many of today's teachers continue to teach mathematics the way they were taught at school (Grootenboer, 2008), thus perpetuating the cycle. ITE offers the opportunity to interrupt the cycle by opening up new possibilities for future prospective teachers.

For many decades, the traditional approach involved teachers (as experts) demonstrating *the* “right way” to solve a problem, followed by extensive practice by students. This meant that teachers could prepare quickly for teaching a particular method they wanted their students to adopt. Reform-based approaches give students the responsibility for coming up with ways of solving problems that make sense to them. Moreover, there is an expectation that students will communicate their thinking and reasoning with each other and their teacher. This process requires teachers to anticipate a range of possible strategies students might use to solve particular problems, and to support their efforts to explain the mental processes they used to reach a solution. This means that teachers need an elaborate and connected understanding of mathematical content knowledge underpinning the key mathematics ideas and processes that they are teaching children at school. The demands on teachers are so much greater now as a consequence of mathematics education reform. However, the rewards in terms of students’ understanding and enjoyment of mathematics promise to be correspondingly greater than previously.

### *Pedagogical Content Knowledge*

Pedagogical Content Knowledge (PCK) has become the focus of much research in mathematics and science education (Shulman, 1986). More recently Ball’s work has been influential in writing about mathematics PCK (e.g., Ball, Thames, & Phelps, 2008). Ball’s model of PCK provides a practice-based theory for “professionally oriented subject matter knowledge in mathematics” (2008, p. 389). Mathematical content knowledge is important for both of the two main components of the model: Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) in mathematics. According to the model, SMK is subdivided into three parts: Common Content Knowledge, Knowledge at the Mathematical Horizon, and Specialised Content Knowledge. Similarly, PCK comprises Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Curriculum.

Researchers investigating the connections between mathematical content knowledge and PCK (e.g., Askew, 2008; Ward & Thomas, 2008) have found no clear linear relationship between these two categories. Having tertiary level mathematics is not necessarily an advantage for primary (mathematics) teachers, although having limited understanding of mathematics may be a major problem for them. Ward and Thomas (2008) found that teachers with low levels of mathematical content knowledge also had low levels of PCK, but those with high levels of content knowledge had a range of scores on the measure of PCK; that is, some teachers with strong content knowledge had low levels of PCK. This evidence supports the claim that a certain threshold of mathematical content knowledge is necessary for good teaching, but being able to meet this requirement is not *sufficient* on its own (e.g., Askew 2008; Sullivan, Clarke, & Clarke, 2009).

Many ITE programs require students on entry to provide evidence of having

achieved a specified level of mathematics knowledge. Despite this, recent studies continue to reveal concerning gaps in prospective teachers' mathematical understandings (Livy & Vale, 2011; Zazkis, Leikin, & Jolfaee, 2011). Currently, New Zealand University Entrance (including 14 credits in mathematics at NCEA<sup>1</sup> Level 1, normally completed in Year 11, two years before leaving secondary school) is taken as providing sufficient evidence of numeracy competency (New Zealand Teachers Council [NZTC], 2010). From 2012, students with Special Admission<sup>2</sup> to university are required by the ITE provider "to meet comparable numeracy requirements as those entering with University Entrance" (NZTC, 2010). Primary teachers in NZ are qualified to teach up to Year 8 (12-13 year-olds), whereas in many countries students this age are taught by specialist mathematics teachers in the secondary system.

### *Teacher Quality at Initial Entry*

Evidence suggests that teacher quality and academic aptitude may have changed substantially over the last few decades, at least in the US and Australia (Hoxby & Leigh, 2004; Leigh & Ryan, 2006). The US study examined a "pull and push" hypothesis that pay parity with males in nonteaching occupations may have drawn able women out of teaching, while lack of remuneration for high achievers has effectively pushed them out of teaching as a possible career. Leigh and Ryan (2006) found that the average person entering teacher education in Australia in the early 1980s was at the 74th percentile for secondary school achievement (Year 9), whereas two decades later the average percentile rank had slipped to 61. The Australian analysis is based on data collected in six longitudinal cohorts of students assessed in literacy and numeracy at age 14, and surveyed about university courses and career choices. Whether students' achievements at age 14 provide a reliable indication of their eventual success at the conclusion of their secondary education might well be questioned. Collins (2011) compared the teacher education practices of recruitment and retention for Singapore, Finland, and South Korea, and contrasted them with those of Australia. All three countries draw their teachers from the top 30 per cent of secondary school graduates, whereas Australia (and the US) have relatively few students from the top one-third applying to become teachers. These three countries also differed from Australia in the high status that the teaching profession is accorded, and the government's role in funding programs and regulating supply to match demand. Although there is no comparable analysis of New Zealand teacher quality and academic aptitude, it seems likely that the patterns are similar to those of Australia and the US.

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1 NCEA stands for National Certificate of Educational Achievement. Students in Year 11 normally complete Level 1, in Year 12 Level 2, Year 13 Level 3.

2 Students over the age of 20 years who have not met the University Entrance requirements while at secondary school are granted Special Admission to university.

### *Assessing Mathematical Proficiency*

Several writers have emphasised the importance of teachers knowing not only *what* understanding and misconceptions to expect from students, but also *why* those misconceptions occur (e.g., Tirosh, 2000). Some researchers have used individual diagnostic interviews to assess the mathematical content knowledge of a small number of ITE students (up to 30; e.g., Tirosh, 2000; Ball, 1990a). Others have used written assessments with larger numbers of ITE students (e.g., 200-300; Ball, 1990b). The written tests used vary according to whether the questions are open-ended or multiple-choice (e.g., Ball, 1990b). Whatever form they take, these assessments are designed to externalize students' knowledge (Shulman, 2000).

Although there is a considerable body of research on the mathematical content knowledge of pre-service elementary teachers, there is little or no information about the mathematical standards required for selection into the ITE program. However, there are indications that the students accepted for some ITE programs in the US are reasonably strong mathematically (e.g., 9 of the 10 prospective primary school teachers interviewed in Ball's 1990a study could successfully solve  $1\frac{3}{4} \div \frac{1}{2}$ ). In contrast, assessment of New Zealand pre-service elementary students showed that less than one third of students entering an ITE program (29% of 159 in 2007, and 18% of 191 in 2008) could solve  $6 \div \frac{1}{2}$  (Young-Loveridge, personal communication).

### *Misconceptions in Mathematics*

Literature on the common errors people make on mathematics tasks suggests that these may reveal important information about the nature of their thinking and problem-solving (Hiebert, et al., 1997). According to Drews (2005), it is important to distinguish between mathematics errors and misconceptions. There are many reasons why errors might be made as part of solution strategies to mathematics problems. For example, some errors are simply the result of carelessness, while others may be a consequence of misinterpreting symbols or text. A person may lack experience or knowledge of the particular topic or concept. Alternatively, the error may be the result of a misconception. Misconceptions differ from other types of errors in that they often reflect some over- or under-generalisation of a rule. According to Swan (2001), learners make similar errors the world over, regardless of what they are taught. Drews (2005, p. 15) contrasts an "avoidance of errors" position for teachers with the view that "identifying and addressing errors" can provide opportunities for improving the mathematical understanding of all learners. However, such a position rests on the assumption that teachers are able to recognise the errors that reflect students' misconceptions in mathematics. Stephens (2006) found that ITE students were not able to anticipate the misconceptions children might have with understanding the equal sign. Evidence suggests that some ITE students make errors that are similar to those of the children they are learning how to teach. The reform-based shift towards encouraging students in thinking, reasoning, and

justifying their problem-solving strategies can help teachers to highlight and address misconceptions in mathematics (e.g., Sullivan et al, 2009). However, it is vital that the teachers recognise when misconceptions occur. Hence ITE programs have an important role to play in helping prospective teachers to become aware of their own understanding and misconceptions so that they are in a position to do this for their students once they become practising teachers.

This paper presents the findings of a study that assessed the numeracy competency and attitudes of 319 ITE students, the majority of whom were enrolled in a three-year primary Bachelor of Teaching program and the remainder in a one-year Graduate Diploma of Teaching.

## Method

The participants in the study included 319 students, 248 undergraduates beginning a three-year Bachelor of Teaching degree and 71 graduates enrolled in a Graduate Diploma of Teaching (see Table 1). The majority of the undergraduates had been awarded University Entrance (UE), one-fifth of them had been given Special Admission, just over one-eighth had come from other tertiary institutions, and the remainder had Discretionary Entry<sup>3</sup>. The graduates constituted one-fifth of the participants overall.

Table 1  
*Number and Percentage of Participants in each category*

| Category of Entry to University    | Number of students | %    |
|------------------------------------|--------------------|------|
| Undergraduates (enrolled in BTchg) | 248                | 77.7 |
| University Entrance                | 150                | 47.0 |
| Special Admission                  | 56                 | 17.6 |
| Other Tertiary Institutions        | 34                 | 10.7 |
| Discretionary Entry                | 8                  | 2.5  |
| Graduates (enrolled in GradDip)    | 71                 | 22.3 |
| Total                              | 319                | 100  |

The students were given the Mathematics Thinking and Reasoning assessment (designed by the authors) during Orientation prior to their ITE program starting (undergraduates) or during their first class (graduates). The assessment consisted of nine tasks, including 2-digit computations and proportional reasoning. Two of the tasks were at Level 3 of *The New Zealand Curriculum*

<sup>3</sup> Students with a strong record of achievement on NCEA Level 2 may apply and be granted Discretionary Entry to university after they have completed Year 12.

([NZC], Ministry of Education, 2007) and seven were at Level 4. The expectation is that students by the end of Year 6 (10- to 11-year-olds) should have achieved Level 3 of the curriculum (and be advanced additive part-whole thinkers), while those at the end of Year 8 (12- to 13-year-olds) should have achieved Level 4 (and be advanced multiplicative part-whole thinkers).

Level 3 tasks were chosen partly to assess content knowledge at this level but also to begin the assessment with easier tasks. Level 4 tasks were chosen from tasks used in previous assessments within the institution, and included fractions, decimals, and proportions because these concepts have been shown previously to be challenging for senior primary school students and ITE students alike (e.g., Ball, 1990a, 1990b; Mays, 2005; Tirosh, 2000). The number of tasks was small and focused only on the number domain because of the limited time available during a busy orientation period (undergraduates) and normal mathematics education class (graduates). Because the tasks were in the number domain, they were easy to align with the levels in the curriculum document (Ministry of Education, 2007).

Students were asked to solve the tasks and show their thinking using words, numbers, and/or pictures/diagrams (calculators were not permitted). Frequencies of correct responses and the most common incorrect responses were calculated.

On the back page of the assessment sheet, was a brief survey asking students to rate their feelings about mathematics at primary and secondary school as well as currently. A four-point Likert-type rating scale was used, ranging from "Really Like/d" to "Really Dislike/d" mathematics, "at primary school", "at secondary school", and "now". Students were also invited to write comments to explain their reasons for the ratings just described. These provided further insights to their experiences and attitudes towards mathematics.

## Results

The results are organised into four major sections. The first focuses on the relationship between mathematical content knowledge and entry status into the ITE program. The second examines the attitudes of students towards mathematics. The third section looks at the relationship between achievement and attitudes towards mathematics. The final section examines students' misconceptions on tasks that significantly differentiated the groups.

### *The Relationship between Mathematical Content Knowledge and Entry Status*

Student performance was analysed as a function of entry status into the ITE program (graduate vs. undergraduate, and for undergraduates, UE vs. no UE). The percentages of students who successfully completed each task according to their entry status are presented in Table 2. Overall, students were more successful on tasks involving whole numbers and addition of decimal fractions. The most challenging tasks were those related to fractional quantities and proportional reasoning.

Students with University Entrance did slightly better than those without University Entrance on most tasks. On average, students with University Entrance attained an average total score half a mark higher than those without ( $M = 6.29$  and  $M = 5.76$  for those with and without UE, respectively; corresponding SDs were 1.71 and 1.80). The biggest differences were evident on the task involving subtraction of decimals with regrouping across place value (66% vs. 55%), addition of common fractions (37% vs. 22%), and converting fractions to percentages (31% vs. 24%) for those with and without UE, respectively. Only on Question 7, addition of common fractions, did undergraduates with UE do significantly better than those without UE [ $\chi^2(1) = 6.09, p < .05$ ].

Table 2 shows that students in the one-year graduate program were more successful than undergraduates on each of the tasks. On average, graduates' total scores were about half a mark higher than those of undergraduates with UE and more than one mark higher than those without UE ( $M = 6.87$  vs. 6.07,  $SD = 1.84$  vs. 1.76). The difference in total score between graduates and undergraduates (overall) was statistically significant [ $t(317) = 3.34, p < .01$ ]. Chi-squared analysis of students' performance on individual tasks revealed a statistically significant difference for Question 4, subtracting decimals with regrouping [ $\chi^2(1) = 5.00, p < .05$ ]. Even larger differences were found on Question 8, 72/90 as a percentage [ $\chi^2(1) = 9.71, p < .01$ ], and Question 7, adding common fractions [ $\chi^2(1) = 14.67, p < .001$ ].

Table 2

*Percentages of students in each group who could do each task*

| Question  | Undergraduates |               |                |               |
|---|----------------|---------------|----------------|---------------|
|   | UE<br>n=150    | no UE<br>n=98 | Total<br>n=248 | Grads<br>n=71 |
| Level 3 Tasks   |                |               |                |               |
| 1 Tama has 64 stickers. He uses 27 on the first day of school. How many does he have left?  | 95             | 96            | 95             | 96            |
| 2 John needs \$403 to buy a stereo. He has saved \$297. How much money does he still need?  | 86             | 85            | 86             | 90            |
| Level 4 Tasks   |                |               |                |               |
| 3 Sue used 8.3 metres of red material and 2.57 metres of blue material to make costumes for the play. How much material did she use altogether? | 91             | 84            | 88             | 92            |

*Continued next page.*



| Question  | Undergraduates |               |                |               |
|---|----------------|---------------|----------------|---------------|
|   | UE<br>n=150    | no UE<br>n=98 | Total<br>n=248 | Grads<br>n=71 |
| 4 Ana bought 4.3 metres of rope to make skipping ropes, but only used 2.89 metres. How much rope was left over?   | 66             | 55            | 62             | 76            |
| 5 If 18 packets each hold 24 felt pens, how many pens is that altogether?   | 65             | 62            | 64             | 69            |
| 6 If 56 plums are shared among 14 people, how many plums will each person get?  | 92             | 91            | 92             | 89            |
| 7 Tama and Karen buy two pizzas. Tama eats $\frac{3}{4}$ of one pizza while Karen eats $\frac{7}{8}$ of the other one. How much pizza do they eat altogether? | 37             | 22            | 32             | 56            |
| 8 If Ben got 72 out of a possible total of 90 marks, what percentage was that?  | 31             | 24            | 28             | 48            |
| 9 Jo spent \$60 on stationery. She got one-third off the original price, because she was a teacher. What was the original price?                              | 65             | 56            | 62             | 72            |

The students were ranked according to their successful performance on the nine tasks. Two undergraduate students answered only one question correctly. Both of these students had met the University Entrance requirements, including the 14 Numeracy Credits. One was successful on the two-digit subtraction and the other divided 56 by 14. Three undergraduate students and three graduates got only two responses correct. One of the undergraduate students had University Entrance, and two had come from another tertiary institution.

Of the six students who had only two correct responses, five were successful on one or both subtraction tasks. None of the students could do the subtraction of decimals with regrouping, multiply  $18 \times 24$ , add common fractions, or convert  $72/90$  to a percentage. Only one student managed to work out the original price before the one-third discount.

Twenty students got a score of three, including 18 undergraduates (7 with UE) and two graduates. Not one of these students could convert  $72/90$  to a percentage. Only one of them was successful in adding common fractions. Another one could multiply  $18 \times 24$ , and two were successful on the decimal subtraction task. Three found the original price before the one-third discount.

Overall, 17 per cent of the students got fewer than half of the tasks correct,

and more than one quarter of undergraduates without UE were in this group (see Table 3). Just over half of the students got between half and almost 90 per cent of the tasks correct. Of the 27 per cent who made no more than one error (90% correct), the biggest proportion were graduates (44%) and the smallest proportion were undergraduates without UE (18%)

Table 3  
Percentages of students in each cohort according to their total score

| Group          | Total Score |   |    |    |        |    |    |    |        |    |    |        | n   |
|----------------|-------------|---|----|----|--------|----|----|----|--------|----|----|--------|-----|
|                | 1           | 2 | 3  | 4  | 1 to 4 | 5  | 6  | 7  | 5 to 7 | 8  | 9  | 8 to 9 |     |
| UE             | 1           | 1 | 5  | 6  | 13     | 18 | 22 | 23 | 63     | 15 | 10 | 25     | 150 |
| no UE          | 0           | 2 | 11 | 13 | 27     | 16 | 24 | 15 | 55     | 11 | 7  | 18     | 98  |
| All undergrads | 1           | 1 | 7  | 9  | 18     | 17 | 23 | 20 | 60     | 13 | 9  | 22     | 248 |
| Graduates      | 0           | 4 | 3  | 4  | 11     | 6  | 20 | 20 | 45     | 24 | 20 | 44     | 71  |
| Overall        | 1           | 2 | 6  | 8  | 17     | 15 | 22 | 20 | 56     | 16 | 11 | 27     | 319 |

### *Attitudes towards Mathematics*

The percentages of students who liked mathematics at primary and secondary school, and currently, are shown in Table 4. Students were most positive about mathematics at primary school (76% of undergraduates and 73% of graduates liked it). Attitude towards mathematics dipped at secondary school, with 46 per cent of the undergraduates and 42 per cent of the graduates indicating that they liked mathematics then. Undergraduates' attitudes towards mathematics had changed little by the time they started in the ITE program, with just under half (45%) of them liking mathematics. Graduates were more positive at the start of the ITE program, with more than half (56%) showing a positive attitude towards mathematics. Students without UE were slightly more positive towards mathematics currently than those with UE (49% vs. 43% liked mathematics currently). This might be a reflection of age and experience, as those with UE were more likely to have been recent school leavers with fresh memories of secondary mathematics, whereas those without UE often had a variety of experiences prior to enrolling in the ITE program. Similarly the graduates would have had a minimum of three years since leaving secondary school. It was interesting to note that it was those with UE who were most positive about mathematics at secondary school (49% liked it), and that between one-third and one-half of the non-UE undergraduates and the graduates (41% and 42%, respectively) liked mathematics at secondary school.

Table 4

*Percentages of students in each cohort according to their attitudes towards mathematics in school*

|                | Undergraduates |              |               | Total        |               |          |
|----------------|----------------|--------------|---------------|--------------|---------------|----------|
|                | UE             | no UE        | Overall       | Graduates    | Overall       | <i>n</i> |
| At Primary     |                |              |               |              |               |          |
| Positive       | 77             | 76           | 76            | 73           | 76            | 239      |
| Neutral        | 6              | 6            | 6             | 3            | 5             | 17       |
| Negative       | 17             | 18           | 18            | 24           | 19            | 60       |
| No of students | <i>n</i> =149  | <i>n</i> =96 | <i>n</i> =245 | <i>n</i> =71 | <i>n</i> =316 |          |
| At Secondary   |                |              |               |              |               |          |
| Positive       | 49             | 41           | 46            | 42           | 45            | 142      |
| Neutral        | 11             | 7            | 9             | 3            | 8             | 25       |
| Negative       | 41             | 52           | 45            | 55           | 47            | 149      |
| No of students | <i>n</i> =150  | <i>n</i> =95 | <i>n</i> =245 | <i>n</i> =71 | <i>n</i> =316 |          |
| Now            |                |              |               |              |               |          |
| Positive       | 43             | 49           | 45            | 56           | 48            | 150      |
| Neutral        | 19             | 18           | 18            | 11           | 17            | 53       |
| Negative       | 39             | 33           | 37            | 32           | 36            | 112      |
| No of students | <i>n</i> =148  | <i>n</i> =96 | <i>n</i> =244 | <i>n</i> =71 | <i>n</i> =315 |          |

### *Relationship between Achievement and Attitudes towards Mathematics*

Table 5 presents the attitudes of students according to their achievement on the assessment tasks. There was a marked tendency for those who scored well (either more than half the tasks correct, or no more than one error) to have a positive view of mathematics. However, even those who got fewer than half of the tasks correct were positive about mathematics at primary school (62%). One-quarter of them were positive about secondary mathematics, and one-fifth (21%) were positive about mathematics currently. Being good at mathematics did not automatically mean that students liked mathematics. More than one-quarter (27%) of high scorers disliked mathematics at secondary school. Chi-Squared analyses showed significant differences in the attitude patterns as a function of performance on the assessment tasks. The effect was relatively small in relation to attitudes to mathematics at primary school [ $\chi^2(4) = 12.97, p < 0.05$ ], but was very strong for attitudes towards mathematics at secondary school [ $\chi^2(4) = 26.78, p < .001$ ] and currently [ $\chi^2(4) = 49.12, p < .001$ ].

Table 5

Percentages of students at each level of achievement according to their attitudes towards mathematics

|                | Total Score  |               |              | Total         | <i>n</i> |
|----------------|--------------|---------------|--------------|---------------|----------|
|                | 1 to 4       | 5 to 7        | 8 to 9       | Overall       |          |
| At Primary     |              |               |              |               |          |
| Positive       | 62           | 74            | 87           | 76            | 239      |
| Neutral        | 6            | 6             | 5            | 5             | 17       |
| Negative       | 32           | 20            | 8            | 19            | 60       |
| No of students | <i>n</i> =53 | <i>n</i> =178 | <i>n</i> =85 | <i>n</i> =316 |          |
| At Secondary   |              |               |              |               |          |
| Positive       | 25           | 41            | 65           | 45            | 142      |
| Neutral        | 6            | 8             | 8            | 8             | 25       |
| Negative       | 69           | 51            | 27           | 47            | 149      |
| No of students | <i>n</i> =52 | <i>n</i> =178 | <i>n</i> =86 | <i>n</i> =316 |          |
| Currently      |              |               |              |               |          |
| Positive       | 21           | 41            | 77           | 48            | 150      |
| Neutral        | 19           | 19            | 11           | 17            | 53       |
| Negative       | 60           | 40            | 13           | 36            | 112      |
| No of students | <i>n</i> =52 | <i>n</i> =177 | <i>n</i> =86 | <i>n</i> =315 |          |

Table 6 presents correlations (*Spearman's rho*) among the attitude ratings at different times and the total score. The high correlation between attitude to mathematics at secondary school and now (.674) indicated that secondary school experiences can be very powerful in shaping students' views about mathematics (between one-third and half of the variance in attitudes at the two time points is shared). This relationship was far stronger than the correlation between current attitude and performance on the assessment tasks (.404) with only 16% of the variance in achievement shared with attitude.

Table 6

Correlations among attitude ratings and total score

| Attitude to Mathematics: | At primary school | At secondary school | Now  | Total score |
|--------------------------|-------------------|---------------------|------|-------------|
| At primary school        | 1                 | .365                | .419 | .351        |
| At secondary school      |                   | 1                   | .674 | .291        |
| Now                      |                   |                     | 1    | .404        |
| Total score              |                   |                     |      | 1           |

### *Low Scorers with Positive Attitudes*

Although in general, students who were successful on the tasks also rated mathematics positively, there were some interesting exceptions to this pattern. For example, 11 students got fewer than half the tasks correct, yet liked mathematics. Their comments revealed information about the reasons for their ratings that is potentially helpful in understanding the complex connections between attitude and achievement. It was evident that these particular students believed that their teachers were crucial in influencing their attitude towards and success in mathematics. The code indicates the course level as undergraduate (UN) and entry category (UE).

I really liked maths at secondary school due to the teaching style of many different maths teachers. They would explain step by step of how to work out the question and we were allowed to use the calculators. (Un1-03 UE)

I did struggle with maths throughout my school years. I enjoyed my 5th form maths the most out of all the years. I had a really good teacher and did really well in the class. I am hoping to improve my maths so that one day I am able to teach it well to my students. I was never very good with percentages or fractions so need to work on them the most. (Un3-14 SA)

I loved maths at high school because I could do it easily. It was my favourite subject and I usually got the best grades in it. My only issue with maths has been the teacher I had in Year 12 who just couldn't teach. I don't remember much about primary school maths or how to do the more interesting stuff in my head (e.g., percentage, long division, division, multiplication, etc) but I can do all the harder stuff. (Un3-22 UE)

In primary school I didn't like maths because I wasn't taught to do maths in an understanding way, and found it very difficult. In secondary school I started to like maths more because it was taught to me in a more fun, enjoyable and understanding way. The teachers were more clear about some of the methods in maths and taught it to me in a way that I could understand. (Un6-19 UE)

One student who liked mathematics valued the opportunities provided by practising exercises from the textbook.

In high school we repeated questions a lot out of text books so we got a lot of practice at understanding the techniques needed to solve the problems, whereas in primary school I hardly ever did practice questions so struggled to remember how to work things out in a test, hence my dislike for maths at primary school. (Un5-27 UE)

Another student explained their ratings by referring to the practical usefulness of mathematics in everyday life.

Maths is something we use all day long without even noticing it. You use speed, distance when walking/driving, percent/\$. (Un6-03 SA)

Several students made comments indicating they were aware of the need to strengthen their mathematics.

I like maths but I'd like to work on my percentages and problem solving, number line as well. (UnT-31 OT)

In my later years of primary I felt as though I wasn't pushed enough to learn things like division, which I still struggle with today. (Un3-18 UE)

### *High Scorers with Negative Attitudes*

Eleven students got almost 90 per cent of the tasks correct (i.e., no more than one error), but were negative in their attitude to mathematics currently. Many of these students referred to particular teachers who had been especially influential in shaping their negative attitudes towards mathematics. Comments about experiences at primary school tended to be positive and these were contrasted with what happened in mathematics classes at secondary school, experiences that seemed to be uppermost in their minds.

At primary school I enjoyed maths because I had good teachers who enjoyed the topic. Once I went to intermediate school, my like for maths went away. This was because my two-year intermediate teacher [Years 7-8] did not enjoy maths and often complained about having to teach it. She also quite often bad-mouthed it, causing me to have a negative view on it. This teacher also relied on maths worksheets and games every day and whilst these were enjoyable, I felt I learnt very little regarding maths those two years because very little teaching took place. By the time I got to high school, I had a very negative view on mathematics. (Un1-25 UE)

I like[d] maths up until I went to secondary school. I found it more difficult to deal with word problems and putting formula into context, and didn't receive a lot of help. (Un4-29 UE)

At primary maths was quite fun, I enjoyed the games and competitions. In my first year of high school I had a good teacher and really enjoyed it. After that I had a few not so good ones and maths stopped making sense. Now basics like these don't come so easy so I don't really enjoy it that much. I enjoy maths when I can do it and it makes sense. (Un5-13 DE)

Several students referred to the use of calculators at secondary school and the lack of understanding they had of the concepts being taught.

I really liked maths at primary because it was explained and taught in different ways. At secondary school I felt that if I didn't get it, that was too bad because we were always moving on to something new in the textbook. At secondary school we were taught how to solve problems on our calculator but I don't know how to get the answer without a calculator. I don't really like maths now because I felt discouraged by it at secondary school and I don't remember a lot of it because once I finished exams I didn't feel there was any need for it. (Un6-08 UE)

In high school we became so reliant on a calculator that I have lost ability to use my own working out which I discovered by doing this test. Maths [has] never been a strong point for me. Really I am a visual and kinesthetic learner which is why I enjoyed it more at primary as we were able to use blocks to work things out and as we got older we were just allowed to use calculators and textbooks which I struggled with actually absorbing what was being taught. (Un5-10 UE)

Other students appeared not to recognise that they had strengths in mathematics, perhaps because they were comparing their success in mathematics with that in literacy, or comparing themselves to people who were good at more challenging mathematics topics. There is also some hint that an emphasis on speed within the mathematics classroom may have had deleterious effects on the attitudes of some students.

I have always found I work better with words than numbers, and can never seem to do quite basic equations in my head very fast. (Un3-11 UE)

### *Misconceptions*

An analysis of common incorrect responses was completed for the cohort on tasks on which there was a statistically significant difference between groups and where at least 2% of students had given the same incorrect response (see Table 7). All of the common mistakes involved responses to tasks from Level 4 of the curriculum. These included adding common fractions, subtracting decimals with regrouping across place value, and converting a fraction to a percentage. For example, subtraction of decimals was challenging for some students who subtracted the smaller decimal part away from the larger part, confusing the subtrahend with the minuend ( $4.3 - 2.89 = 1.59$ ). More undergraduates without UE (12%) made this mistake compared to those with UE (8%), but few graduates made this error (4%).

Addition of common fractions also posed challenges for many of the students. Two-thirds of the students had problems adding  $\frac{3}{4}$  and  $\frac{7}{8}$ . The most common mistake was to convert  $\frac{3}{4}$  to  $\frac{6}{8}$  then add both the numerators and denominators to get an answer of  $\frac{13}{16}$ . This mistake was made by almost as many of those with UE as those without UE (15% vs. 19%). Some other students did not find an equivalent fraction for  $\frac{3}{4}$ , instead immediately adding both numerators and denominators to get an answer of  $\frac{10}{12}$  (19% of those with UE; 26% of those without UE). These mistakes suggest students were either simply executing a mis-learned procedure and not paying attention to the meaning of the problem, or treating the numerator and denominator as separate numbers. The students neglected to apply 'number sense'. A sense of the size of the magnitude of the fraction in relation to the whole would have helped them realise that both  $\frac{3}{4}$  and  $\frac{7}{8}$  are close to one whole, so the answer had to be greater than one. Just over one-quarter of the students (28% overall; 31% of those with UE and 24% of those without UE) could convert 72 out of 90 to a percentage by noticing that 72 and 90 are both multiples of 9, so the fraction could easily be simplified to  $\frac{8}{10}$  and then

converted to 80%. Alternatively, students could have noticed that every nine marks was worth 10% and calculated how many nines are in 72, or used benchmarks such as half ( $45 = 50\%$ ), quarter ( $22.5 = 25\%$ ), and one fifth of one quarter ( $4.5 = 5\%$ ), then added the parts together to get 72 marks and 80%. Quite a number of students wrote the formula (or part of it) to calculate  $72 \div 90 \times 100$ , indicating dependence on algorithmic procedures.

Table 7

*Percentages of students who made common errors on selected tasks*

| Question   | Undergraduates      |                       |                        | Grads<br><i>n</i> =71 |
|--|---------------------|-----------------------|------------------------|-----------------------|
|  | UE<br><i>n</i> =150 | no UE<br><i>n</i> =98 | Total<br><i>n</i> =248 |                       |
| 4 Ana bought 4.3 metres of rope to make skipping ropes, but only used 2.89 metres. How much rope was left over?<br>(Correct Ans: 1.41m)  | 66                  | 55                    | 62                     | 76                    |
| Ans: 1.14  | 2                   | 5                     | 3                      | 3                     |
| Ans: 1.59  | 8                   | 12                    | 10                     | 4                     |
| Ans: 2.59  | 6                   | 2                     | 4                      | 1                     |
| No Answer Given  | 5                   | 7                     | 6                      | 7                     |
| 7 Tama and Karen buy two pizzas. Tama eats $\frac{3}{4}$ of one pizza while Karen eats $\frac{7}{8}$ of the other one. How much pizza do they eat altogether?<br>(Correct: $\frac{13}{8}$ or 1 and $\frac{5}{8}$ ) | 37                  | 22                    | 32                     | 56                    |
| Ans: $\frac{10}{12}$   | 19                  | 26                    | 21                     | 3                     |
| Ans: $\frac{13}{16}$   | 15                  | 19                    | 17                     | 13                    |
| No Answer Given  | 5                   | 8                     | 6                      | 4                     |
| 8 If Ben got 72 out of a possible total of 90 marks, what percentage was that?<br>(Correct Ans: 80%)   | 31                  | 24                    | 28                     | 48                    |
| Ans: 0.648 or $.72 \times 90$ or 64.8  | 6                   | 6                     | 6                      | 4                     |
| Ans: 0.81  | 3                   | 3                     | 3                      | 0                     |
| Ans: 0.82 or 82  | 11                  | 14                    | 12                     | 4                     |
| Ans: $\frac{72}{90}$ or $\frac{72}{90} \times 100$   | 15                  | 10                    | 13                     | 0                     |
| No Answer Given  | 21                  | 22                    | 22                     | 31                    |



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## Discussion

Like other studies that have investigated the mathematical content knowledge of pre-service teachers (e.g., Callingham & Beswick, 2011), this study found that mathematical knowledge was relatively weak. Many students used algorithmic procedures to calculate answers. They did not use knowledge of number properties to find common factors or apply the same operation to both dividend and divisor (e.g., for  $56 \div 14$ ), instead resorting to drawing tally marks and counting groups of 14.

Many participants were unable to use calculation strategies based on number sense to add common fractions ( $\frac{3}{4} + \frac{7}{8}$ ) (68%), or convert a fraction (72 out of 90) to a percentage (72%). These tasks were within the curriculum levels for which they were preparing to teach (Levels 3-4, 9-13-year-olds). Over one third of participants added both numerators and denominators for addition of common fractions. The performance of graduates overall was better than undergraduates. On the other hand, some graduates made mistakes similar to those of undergraduates, also adding across numerators and denominators, but usually after converting  $\frac{3}{4}$  to  $\frac{6}{8}$  before adding on  $\frac{7}{8}$ . The findings challenge assumptions currently being made by ITE programs that UE (including the 14 Numeracy credits) provides a reliable indicator of students' numeracy competencies. Instead the findings indicate the need for systematic assessment on entry into ITE, coupled with careful monitoring and support to ensure students reach satisfactory levels of mathematical content knowledge by graduation. These findings are interesting in view of the proposition that ITE become a postgraduate only program (New Zealand Government, 2010).

It is interesting to reflect on the absolute levels of achievement of students taking the test. White et al. (2005/2006) bemoaned the fact that 13% of their participants scored less than 50% on their assessment tasks (basic number skills at early secondary school level). However, in this study, over one quarter (27%) of undergraduates without UE got less than 50%, and the tasks used were probably easier.

Compared to Biddulph's (1999) findings, the ITE students in the present study were more positive (48% vs. 25%) and less negative (36% vs. 59%) in their attitudes towards mathematics (noting that Biddulph's two cohorts are averaged here). Nevertheless, the finding that fewer than half the students liked mathematics is still an issue of considerable concern. Aligning these results with others on students nearing the end of their ITE program (57% were positive and only 16% were negative; see Young-Loveridge, 2010) suggests that by the end of three-year training, students become more positive about mathematics. Another reason for optimism about the attitudes of ITE students is that final-year students were more positive about the prospect of teaching mathematics than about mathematics generally (67% vs. 57%). Similar numbers of final-year students, however, were negative about mathematics and about teaching mathematics (16% and 17%). Finding one-third of prospective teachers who dislike mathematics at the beginning of their training and one-fifth who are negative

about mathematics and teaching mathematics at the end of their training is still far from satisfactory. This finding challenges the idea that some threshold level of mathematical content knowledge alone should be the only factor considered in relation to teacher quality. This is consistent with the arguments proposed by the AITSL (2011) that teacher quality is determined by three variables: mathematical content knowledge, attitude towards teaching mathematics, and understanding of mathematics pedagogy.

It must be acknowledged that the nine tasks used to assess the students here represent only a small part of the mathematics domain. These concepts, however, are foundational for many ideas across the mathematics curriculum, and are a key component of mathematical content knowledge. Other research on mathematical content knowledge has had a similar focus (e.g., Ball, 1990a, 1990b, Biddulph, 1999; Tirosh, 2000), and Biddulph's (1999) study used only six tasks.

This study shows that students enter ITE with minimal levels of mathematical content knowledge. The undergraduate participants in this study had three years in which to strengthen their mathematics. Current regulations require them to complete 72 hours of compulsory papers (university subjects) in mathematics, including methods. These papers are completed halfway through their program, leaving eighteen months in which many do no further mathematics before graduation. Given the areas of weakness in mathematical content knowledge identified here, it is questionable whether their performance can be brought to an acceptable level. Currently students are not assessed before graduation to ensure they meet numeracy competency requirements. However, they can choose to strengthen their understanding in mathematics through an optional third-year paper.

Overall, students' attitudes towards mathematics were not enthusiastic, with fewer than half of the students liking mathematics, and over one third disliking mathematics currently. Just under half the students disliked mathematics at secondary school, and much of this negativity seems to have continued to the present day, possibly because a substantial proportion of undergraduates had only recently left secondary school. It is acknowledged that students' ratings of attitude might have been adversely affected by having just completed the assessment tasks. This applies to other research in which attitudes and content knowledge were assessed together (e.g., Biddulph, 1999; Young-Loveridge, 2010).

It is interesting to note that many participants held their teachers responsible for positive as well as negative ratings of mathematics, consistent with Hattie's (2009) conclusion that teachers have a substantial impact on their students. The students also attributed low levels of confidence and difficulties in mathematics to their teachers at primary and/or secondary school. Some recognised the importance of connections between mathematics and real-life situations, as well as the importance of understanding what one is learning about. Baseline data signalling fear or dislike of mathematics provides a sound rationale for teacher educators to take seriously the affective dimension in the learning and teaching of mathematics. ITE students' attitudes and beliefs can be addressed, given a reasonable period of time (2-3 years; see McGinnis et al., 2002).

Diagnosing the mathematical misconceptions of adults preparing to be primary teachers provides important and interesting insights that need to be taken into account in ITE programs (Tirosh, 2000). Our ITE students need to be aware not only of misconceptions in their own thinking, but also recognise these in their future pupils. For teacher educators, the challenge is to break the cycle. Swan (2001) suggests that "cognitive conflict" can be used to shift learners' understanding through a series of design principles, including: initial assessment, followed by tasks designed to provoke conflict discussion, leading to resolutions and the formulations of new concepts and methods.

The misconceptions identified through written tasks revealed what was inside the learners' heads. Shulman (2000) argues that the first "pedagogical challenge" for teachers is "to bring what is inside, out: to make the internal external, to make the private public, to make the implicit explicit" (p. 133). This was possible because the tasks were open-ended rather than multiple-choice. Expecting learners to explain their thinking is vital if teachers are to be able to address their learning needs effectively. This strategy could strengthen the discourse between teachers and students in ITE programs.

It is extremely difficult to change the ways mathematics is taught and learned (Anthony & Hunter, 2005; Lamon, 2007). Teacher educators need to be sensitive to the dilemma of addressing the limited SMK of pre-service education students, including their misconceptions, while at the same time focusing on the complexities of becoming effective teachers of these very same concepts. An additional challenge is dealing with the negative attitudes towards mathematics that have been shaped by previous experiences of mathematics in school and beyond (Ball, Lubienski, & Mewborn, 2001).

If a certain threshold of discipline knowledge in mathematics is necessary for good teaching, then it is vital that institutions assess prospective teachers to ascertain the extent of that knowledge and identify particular areas that may need to be further strengthened. The findings suggest that the use of numeracy assessment tasks to reveal important misconceptions could be helpful in determining the extent to which students are likely to meet a threshold level of proficiency in mathematics. If a rigorous selection process includes assessment of students' mathematical understandings, then surely it should improve teacher quality (McArdle, 2010). Attracting the highest calibre of young people to the teaching profession is the rationale behind the New Zealand government's latest plan to shift the focus of pre-service teacher education to a postgraduate qualification as a minimum requirement for all trainee teachers (NZ Government, 2010). Teachers who have sound mathematical content knowledge and a positive attitude towards mathematics could potentially become competent and confident teachers who engage learners in meaningful and worthwhile mathematics.

## References

- Anthony, G., & Hunter, R. (2005). A window into mathematics classrooms: Traditional to reform. *New Zealand Journal of Educational Studies*, 40, 25-43.
- Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs designed to develop that knowledge. In P. Sullivan & T. Woods (Eds.), *The international handbook of mathematics teacher education: Vol. 1* (pp. 13-35). Rotterdam, The Netherlands: Sense Publishers.
- Australian Council of Deans of Education [ACDE] (2012). *ACDE Report to AITSL on Standards 3.1 and 3.2*. Retrieved from: [http://www.acde.edu.au/pages/images/AITSL\\_Standard\\_3%201\\_\\_3%202\\_Final\\_Report\\_from\\_ACDE\\_25\\_Jun\\_12.pdf](http://www.acde.edu.au/pages/images/AITSL_Standard_3%201__3%202_Final_Report_from_ACDE_25_Jun_12.pdf)
- Australian Institute for Teaching and School Leadership [AITSL] (2012). *National professional standards for teaching*. Melbourne, Australia: Education Services Australia. Retrieved from: <http://www.teacherstandards.aitsl.edu.au/Standards/AllStandards>
- Ball, D. L. (1990a). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, 21(2), 132-144.
- Ball, D. L. (1990b). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal*, 90(4), 449-466.
- Ball, D. L., Lubienski, S., & Mewborn, D. S. (2001). Mathematics. In V. Richardson (Ed.), *Handbook of research on teaching*. Washington, D.C.: American Educational Research Association.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.
- Biddulph, F. J. (1999). The legacy of schooling: Student teachers' initial mathematical feelings and competence. *Mathematics Teacher Education and Development*, 1, 64-71.
- Callingham, R. & Beswick, K. (2011). Measuring pre-service teachers' knowledge of mathematics for teaching. In J. Wright (Ed.), *Proceedings of the annual conference of the Australian Association for Research in Education*. Retrieved from: <http://www.aare.au/11pap/papers.pdf/aarefinal00497.pdf>
- Collins, R. (2011). Quality teachers: Simply the best? *Teacher: The National Education Magazine*, 218, 6-14.
- Commonwealth of Australia (2008). *National numeracy review report*, May 2008. Canberra, ACT: Author.
- Department for Education (2010). *The importance of teaching: The schools white paper 2010*. London, UK: Author. Retrieved from: <http://www.official-documents.gov.uk>
- Drews, D. (2005). Children's mathematical errors and misconceptions: Perspectives on the teacher's role. In A. Hansen (Ed.), *Children's errors in mathematics: Understanding common misconceptions in primary schools* (pp. 14-21). Exeter, UK: Learning Matters.
- Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education*, 11, 479-497.
- Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. London, UK: Routledge.
- Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., ... Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.
- Hoxby, C. M., & Leigh, A. (2004). Pulled away or pushed out? Explaining the decline of teacher aptitude in the United States. *Teacher Quality*, 94(2), 36-240.
- Lamon, S. (2007). Rational numbers and proportional reasoning: Towards a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-667). Charlotte, NC: Information Age Publishing.

- Leigh, A., & Ryan, C. (2006). *How and why has teacher quality changed in Australia?* The Australian National University Centre for Economic Policy Research Discussion Paper no. 534. Retrieved from: <http://andrewleigh.org/research.htm#EconomicsOfEducation>
- Livy, S., & Vale, C. (2011). First year pre-service teachers' mathematical content knowledge: Methods of solution for a ratio question. *Mathematics Teacher Education and Development*, 13(2), 22-43.
- Lomas, G., Grootenboer, P., & Attard, C. (2012). The affective domain and mathematics education. In B. Perry, T. Lowrie, T. Logan, A. MacDonald, & J. Greenlees (Eds.), *Research in mathematics education in Australasia: 2008-2011* (pp. 23-37). Rotterdam, The Netherlands: Sense Publishers.
- Mays, H. (2005) Mathematical knowledge of some entrants to a pre-service education course, in M. Coupland, J. Anderson, & T. Spencer (Eds.), *Making mathematics vital: (Proceedings of the 20th biennial conference of the Australian Association of Mathematics Teachers*, pp. 186-193). Adelaide, SA: AAMT.
- McArdle, F. (2010). Preparing quality teachers: Making learning visible. *Australian Journal of Teacher Education*, 35(8), 60-78.
- McGinnis, J. R., Kramer, S., Shama, G., Graeber, A. O., Parker, C. A., & Watanabe, T. (2002). Undergraduates' attitudes and beliefs about subject matter and pedagogy measured periodically in a reform-based mathematics and science teacher preparation program. *Journal of Research in Science Teaching*, 39(8), 713-737.
- Ministry of Education (2007). *The New Zealand curriculum*. Wellington: Author.
- New Zealand Government (2010). *A vision for the teaching profession: Education workforce advisory group report to the Minister of Education*. Wellington, NZ: Author.
- New Zealand Teachers Council (NZTC) (2010). *Approval, review and monitoring processes and requirements for initial teacher education programmes*. Wellington, NZ: Author. Retrieved on 16 November 2011 from: <http://www.teacherscouncil.govt.nz/te/iteproviders.stm>
- Organisation for Economic & Cooperative Development [OECD] (2005). *Teachers matter: Attracting, developing and retaining effective teachers*. Paris, France: Author.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15, 4-14.
- Shulman, L. S. (2000). Teacher development: Roles of domain expertise and pedagogical knowledge. *Journal of Applied Developmental Psychology*, 21(1), 129-135.
- Skemp, R. R. (1972). *The psychology of learning mathematics*. Harmondsworth, UK: Penguin.
- Southwell, B., White, A., Way, J., & Perry, B. (2005). Attitudes versus achievement in pre-service mathematics teacher education. In *Proceedings of the annual conference of the Australian Association for Research in Education*, Melbourne, December 2004. Melbourne: AARE. Retrieved on 23 January 2012 from: <http://www.aare.edu.au/05pap/sou05705.pdf>
- Stephens, A. C. (2006). Equivalence and relational thinking: Preservice elementary teachers' awareness of opportunities and misconceptions. *Journal of Mathematics Teacher Education*, 9, 249-278.
- Sullivan, P., Clarke, D., & Clarke, B. (2009). Converting mathematics tasks to learning opportunities: An important aspect of knowledge for mathematics teaching. *Mathematics Education Research Journal*, 21(1), 85-105.
- Swan, M. (2001). Dealing with misconceptions in mathematics. In P. Gates (Ed.), *Issues in mathematics teaching* (pp. 147-165). London, UK: RoutledgeFalmer.
- Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education*, 31(1), 5-25.

- Ward, J. & Thomas, G. (2008). Does teacher knowledge make a difference? *Findings from the New Zealand Numeracy Development Projects 2007* (pp. 50-60). Wellington: Ministry of Education.
- Watson, J. M. (2011). Personal and professional numeracy: A unit for preservice teachers at the University of Tasmania. *Numeracy: Advancing Education in Quantitative Literacy*, 4(1). Retrieved from:  
<http://services.bepress.com/numeracy/vol4/iss1/art2>
- White, A.L., Way, J., Perry, B., & Southwell, B. (2005/2006). Mathematical attitudes, beliefs and achievement in primary pre-service teachers. *Mathematics Teacher Education and Development*, 7, 33-52.
- Young-Loveridge, J. (2010). Two decades of mathematics education reform in New Zealand: What impact on the attitudes of teacher education students. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia) (pp. 705-712). Fremantle, WA: MERGA.
- Zazkis, R., Leikin, R., & Jolfaee, S. C. (2011). Contributions of "mathematics for elementary teachers" courses to teaching: Prospective teachers' views and examples. *Mathematics Teacher Education and Development*, 13(2), 3-21.
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